

**Calculators, Mobile telephones and Pagers ARE NOT ALLOWED.**

Answer all of the following questions.

1. (a) (1 point) Show that  $\log_{\pi} \sqrt{\pi + \pi^{-1} - 2} = -\frac{1}{2} + \frac{2}{\ln \pi} \ln(\pi - 1)$ .

(b) (1 point each) Compute the exact value of

(i)  $\sinh(\ln 2) - \sqrt[3]{\ln e^8}$

(ii)  $\cos^{-1}(\cos(\frac{5\pi}{4}))$

(iii)  $\frac{e^{\frac{1}{2}} + \sin^{-1}(\frac{1}{2})}{e^{\ln(\frac{\pi}{8} + \sqrt{e})}}$

2. (4 points) Evaluate  $\lim_{x \rightarrow \infty} (1 + \tan^{-1}(e^{-x}))^{e^x}$

3. (5 points) Let  $f(x) = \sin^{-1}(\sqrt{\ln x}) - 5$ .

(a) Find the domain of  $f$ . Show that  $f$  is one-to-one on its domain.

(b) State the domain and range of  $f^{-1}$  and compute  $f^{-1}(x)$ .

4. Evaluate the following integrals (5 points each)

(a)  $\int (\sin 2x)^2 \cos^3 x \, dx$

(b)  $\int \frac{x e^x}{(x+1)^2} \, dx$

(c)  $\int \frac{\cos x}{(\sin^2 x + 8 \sin x + 7)^{\frac{3}{2}}} \, dx$

(d)  $\int_1^{\infty} \frac{dx}{x+x^3}$ , if it converges.

5. (5 points) If the curve  $C$  is given parametrically as

$$x(t) = \sin^{-1}(e^{-t}), \quad y(t) = \sec^{-1}(e^t), \quad 0 \leq t \leq \ln \sqrt{10},$$

then find the length of  $C$ .

6. (4 points) Find the area inside the graph of  $r = 3 \sin \theta$  and outside the graph of  $r = 2 - \sin \theta$ .

7. (4 points) Find the equation and sketch the graph of the hyperbola whose vertices are  $V_1(1, 2)$  and  $V_2(5, 2)$  and passes through the point  $P(0, 1)$ .

8. (4 points) Find the equation of the plane containing the line  $\frac{x+2}{3} = \frac{y-4}{2} = \frac{z-3}{-1}$  and perpendicular to the plane  $x + 2y + z = 10$ .